Higher-Order Asymptotic Methods in Statistics and Econometrics

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1 Abstract

The goal of this tutorial is to present the fundamental ideas and tools of higher-order asymptotic methods and their use in statistics and econometrics.

Classical statistics and econometrics typically rely on assumptions made on the structural and the stochastic parts of the model and on optimal procedures derived under these assumptions. Standard examples are least squares estimators in linear models and their extensions, maximum likelihood estimators and the corresponding likelihood-based tests, and GMM techniques in econometrics. Inference is typically carried out by means of standard (first-order) asymptotic theory. However, this can lead to inaccurate p-values and confidence intervals when the sample size is moderate to small or when probabilities in the tails are required (and in some cases, such as e.g. the estimation of VaR, both are required).

Several different techniques have been devised to improve first-order asymptotic approximations and to derive more accurate p-values and confidence intervals. They include Edgeworth expansions, Bartlett's corrections, bootstrap methods, empirical likelihood techniques, and saddlepoint approximations. After a brief review of standard asymptotic theory, we will discuss these methods by focusing on the underlying ideas, their properties, the links among them, and by providing numerical examples.

A special chapter will be devoted to saddlepoint methods which provide approximations of the distribution of general estimators and test statistics with an accuracy given by a relative error of order $O(1/n)$. This property leads to very accurate approximations for moderate to small sample sizes and far out in the tails of the distribution.

2 Contents

3 Selected References

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